NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 358

TEMPERATURE COEFFICIENT OF THE MODULUS OF RIGIDITY OF AIRCRAFT INSTRUMENT DIAPHRAGM AND SPRING MATERIALS

By W. G. BROMBACHER and E. R. MELTON

1930

For sale by the Superintendent of Documents, Washington, D. C. - - - - - - - - - - - - - - - Price 10 cents

AERONAUTICAL SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Symbol	Unit	Symbol
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of one kilogram	kg	weight of one pound	lb.
Power	P	kg/m/s		horsepower	hp
Speed		{km/hr	k. p. h.	mi./hr.	m. p. h.
		{m/s	m. p. s.	ft./sec.	f. p. s.

2. GENERAL SYMBOLS, ETC.

W, Weight, $=mg$

g, Standard acceleration of gravity $=9.80665$ m/s$^2=32.1740$ ft./sec.^{2}

m, Mass, $=\frac{W}{g}$

ρ, Density (mass per unit volume).

Standard density of dry air, 0.12497 (kg-m^{-4} s^2) at 15° C and 760 mm $=0.002378$ (lb.-ft.$^{-4}$ sec.^{2}).

Specific weight of "standard" air, 1.2255 kg/m$^3=0.07651$ lb./ft.^{3}

mk^2, Moment of inertia (indicate axis of the radius of gyration, k, by proper subscript).

S, Area.

S_w, Wing area, etc.

G, Gap.

b, Span.

c, Chord length.

b/c, Aspect ratio.

f, Distance from C. G. to elevator hinge.

μ, Coefficient of viscosity.

3. AERODYNAMICAL SYMBOLS

V, True air speed.

q, Dynamic (or impact) pressure $=\frac{1}{2}\rho V^2$

L, Lift, absolute coefficient $C_L=\frac{L}{qS}$

D, Drag, absolute coefficient $C_D=\frac{D}{qS}$

C, Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

R, Resultant force. (Note that these coefficients are twice as large as the old coefficients L_C, D_C.)

i_w, Angle of setting of wings (relative to thrust line).

i_t, Angle of stabilizer setting with reference to thrust line.

γ, Dihedral angle.

$\rho\frac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi./hr. normal pressure, 0° C: 255,000 and at 15° C., 230,000;

or for a model of 10 cm chord 40 m/s, corresponding numbers are 299,000 and 270,000.

C_p, Center of pressure coefficient (ratio of distance of C. P. from leading edge to chord length).

β, Angle of stabilizer setting with reference to lower wing, $=(i_t-i_w)$.

α, Angle of attack.

ϵ, Angle of downwash.

REPORT No. 358

TEMPERATURE COEFFICIENT OF THE MODULUS OF RIGIDITY OF AIRCRAFT INSTRUMENT DIAPHRAGM AND SPRING MATERIALS

By W. G. BROMBACHER and E. R. MELTON
Bureau of Standards

116354—30

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

NAVY BUILDING, WASHINGTON, D. C.

(An independent Government establishment, created by act of Congress approved March 3, 1915, for the supervision and direction of the scientific study of the problems of flight. Its membership was increased to 15 by act approved March 2, 1929 (Public, No. 908, 70th Congress). It consists of members who are appointed by the President, all of whom serve as such without compensation.)

REPORT No. 358

TEMPERATURE COEFFICIENT OF THE MODULUS OF RIGIDITY OF AIRCRAFT INSTRUMENT DIAPHRAGM AND SPRING MATERIALS

By W. G. Brombacher and E. R. Melton

SUMMARY

Experimental data are presented on the variation of the modulus of rigidity in the temperature range −20 to +50° C. of a number of metals which are of possible use for elastic elements for aircraft and other instruments. The method of the torsional pendulum was used to determine the modulus of rigidity and its temperature coefficient for aluminum, duralumin, Monel metal, brass, phosphor bronze, coin silver, nickel silver, three high carbon steels, and three alloy steels. The temperature coefficient m is defined by the relation

$$m=\frac{1}{G_0}\frac{\mathrm{d}G}{\mathrm{d}T}$$

in which G and G_0 are the moduli of rigidity at the temperatures T° C. and 0° C. The differential $\mathrm{d}G/\mathrm{d}T$ *was found to be a constant except for two metals. The effect of heat treatment on m was determined for a number of the materials. It was observed that tensile stress affected the values of the modulus by amounts of 1 per cent or less.*

INTRODUCTION

Aircraft instruments must operate satisfactorily at temperatures which may vary between −50° C. and +50° C. Satisfactory operation means not only that the instruments function, but that their indications be accurate within tolerances. Thus current military specifications provide for tests at instrument temperatures of −35° C. and +45° C. It is well known that the indication of an instrument depending upon the deflection of an elastic element, such as a diaphragm or spring, varies with temperature. In properly designed instruments the major part of this variation is due to the variation of the elastic moduli with temperature. It is obvious in considering the performance of aircraft instruments that data are desirable on the effect of temperature on the elastic moduli of the materials commonly used for elastic elements.

There are no data on many of the commonly used alloys. Such data as are available are of little use in instrument work either because they are not sufficiently accurate or because they do not cover the required temperature range. A very important point is that for the most part investigators have secured data on annealed specimens, while the elastis elements of instruments consist of hardened materials. It is interesting to note that values of the temperature coefficients of elasticity are not given in current handbooks of physical data.

The Bureau of Standards, with the financial support and cooperation of the National Advisory Committee for Aeronautics, has undertaken an investigation of the effect of temperature on the elastic properties of metals and alloys which may be useful in the field of aeronautic instruments. The objects of the research are as follows:

(*a*) To determine the temperature effect on the elastic moduli of the various common diaphragm and spring materials in the temperature range from −50° to +50° C.

(*b*) To investigate materials in which the temperature coefficient of elasticity is relatively small or anomalous.

(*c*) To determine the effect of stress, heat treatment, and other factors on the elastic moduli and the temperature coefficient of elasticity.

Item (*b*) on the above program is of some importance in view of the not inconsiderable change in indication of instruments with elastic elements in the temperature range experienced in aircraft. Attention should be called to the extensive work on this phase of the problem by Guillaume and Chevenard (References 4 and 7). As a result of Guillaume's work on alloys of the nickel-iron series one was found which has a very low temperature coefficient of elasticity (both for the Young's and the rigidity modulus) at room temperatures. This alloy is known as elinvar. Its possible use in aircraft instruments depends on knowledge of its behavior in the above-mentioned temperature range, together with its resistance to corrosion, its ease of mechanical working, and its elastic properties which should be at least as good as those of alloys now commonly used. Also there must be a reasonable prospect that eventually it will be easily obtainable. The information now available is not sufficient to determine its usefulness in aircraft instruments.

This paper gives the results of experimental work thus far completed at the Bureau of Standards. The data include (*a*) values of the temperature coefficient of the modulus of rigidity of a number of common

materials, (*b*) values of the rigidity modulus at 0° C., and (*c*) to a limited extent the effect of tension, and (*d*) of heat treatment. The temperature range in which data were obtained was from −25° to +50° C. The torsion pendulum method was adopted, and since this method is not to be used in the future experiments, it is considered advisable to report on the results which have already been obtained.

The experiments are being continued using the deflection of helical springs as the criterion instead of the period of a torsion pendulum. The new method will enable better temperature control, as the springs can be tested in a liquid bath and, more important, the determination of the temperature coefficient for both the rigidity and Young's modulus can be made on the same specimen. The method also gives promise of greater rapidity in obtaining data.

THEORY

Neglecting the effect of internal and air damping, the period P of a torsion pendulum with a round wire is given by the equation

$$P^2 = \frac{8\pi LI}{Gr^4} \tag{1}$$

in which G = the modulus of rigidity,
r = the radius of the wire,
I = the moment of inertia of the oscillating system,
L = the length of the wire.

In accordance with equation (1) define

$$G_T = \frac{8\pi L_T I_T}{r^4_T P_T^2} \quad \text{and} \quad G_0 = \frac{8\pi L_0 I_0}{r_0^4 P_0^2} \tag{2}$$

where the subscripts refer to the temperature (degrees Centigrade) at which the quantities are measured.

In the determination of the modulus the geometrical dimensions of the torsion pendulum at 20° C., approximately, were used. This defines the following equations:

$$G_T' = \frac{8\pi L_{20} I_{20}}{r_{20}^4 P_T^2} \quad \text{and} \quad G_0' = \frac{8\pi L_{20} I_{20}}{r_{20}^4 P_0^2}. \tag{3}$$

The temperature coefficient of the modulus of rigidity is defined by

$$m = \frac{1}{G_0}\frac{dG_T}{dT}. \tag{4}$$

Using the terms defined in equation (3),

$$m' = \frac{1}{G_0'}\frac{dG_T'}{dT}. \tag{5}$$

It is apparent that m does not differ very much from m'. The relation between m and m' is found as follows. Dividing equation (4) by equation (5),

$$\frac{m}{m'} = \frac{G_0'}{G_0}\frac{\frac{dG_T}{dT}}{\frac{dG_T'}{dT}}. \tag{6}$$

Now

$$G_0' = G_0[1 + (2a - 3b)\ 20] \tag{7}$$

where a and b are the linear thermal coefficients of expansion of the weights and of the wire specimen, respectively. Similarly

$$G_T = G_T'\ [1 + (2a - 3b)\ (T - 20)] \tag{8}$$

From equation (7) it follows that

$$\frac{G_o'}{G_o} = 1 + (2a - 3b)20 \tag{9}$$

and from equation (8),

$$\frac{\frac{dG_T}{dT}}{\frac{dG_T'}{dT}} = \frac{2a-3b}{m'}\frac{G_T'}{G_0'} + [1 + (2a - 3b)(T - 20)] \tag{10}$$

Multiplying equation (9) by equation (10) and neglecting relatively higher order quantities there is obtained the desired relation:

$$m = m' + (2a - 3b) \tag{11}$$

As will be shown later, the effect of the damping on the values of the modulus of rigidity and the temperature coefficient is negligible under our conditions of experimentation. This conclusion agrees with that of other investigators, notably Horton (Reference 1).

Modulus of Rigidity by the Deflection Method.—The modulus of rigidity was also determined in most cases by the deflection method using the apparatus shown in Figure 3, in order to check the values obtained by the torsional pendulum. The modulus of rigidity G is given by the following expression

$$G = \frac{2lM}{\pi r^4 a} \tag{12}$$

in which l and r are respectively the length and radius of the wire specimen and M the torque required to produce the angular deflection a.

DESCRIPTION OF SAMPLES

Samples of wire as straight as possible were secured of a number of materials. All except a sample of annealed Monel metal were in a hard-drawn condition. The aluminum sample (2 S 1/2 H) was commercially pure, so fabricated as to be one-half hard. The duralumin sample (17 S–T) was commercial heat treated. The oil-tempered steel wire and piano wire samples were furnished by the manufacturer, heat treated in the usual manner for use in making helical springs.

Chemical Composition.—Analyses of the samples were made by the Chemistry Division of the Bureau of Standards. The results are given in Table I.

The percentage aluminum in the aluminum and duralumin samples was determined by the difference between 100 per cent and the percentage of the other constituents for which the samples were analyzed.

Heat Treatments.—A number of the specimens were heat treated in the manner shown in Table II. All of the heat treatments listed for a given material were made on the same pieces of wire. Between heat treatments the wire was replaced in the torsion pendulum and the usual data obtained.

The wires were quenched in water after being heated by passing an electrical current through them. They were mounted horizontally without thermal insulation and under a tension sufficient to keep them straight. In order to maintain the temperature of the wire up to the point of immersion, the apparatus was arranged so that the hot wire could be rotated into the tank of water during which operation a sliding switch automatically cut off the heating current. (See fig. 2.) The temperature was determined by an optical pyrometer.

The drawing or tempering of the specimens was in all cases carried out in an inclosed type of electric furnace. This consisted of an inner porcelain tube of small bore which was placed in a small tubular furnace. The temperature of the furnace could be regulated to within 4° C. It was measured by one thermocouple placed at the mid-point of the interior of the furnace. It was found by experiment that the temperature of the furnace had practically the same value over its entire length 15 minutes after stabilizing the heating current. The temperature as indicated by the thermocouple could be kept constant within 3° C. Care was taken not to exceed the desired temperature. The wires were held at the tempering temperature for not more than one-half hour, and were left in the furnace until it had cooled to room temperature.

TABLE I.—CHEMICAL COMPOSITION OF SAMPLES

[When zero amount of an element is given, it means none was detected]

(*a*) *Nonferrous*

[Elements in per cent]

No.	Samples	Silver	Copper	Zinc	Aluminum	Lead	Tin	Nickel	Silicon	Iron	Phosphorus	Manganese	Magnesium
21	Aluminum	----------	0	0	[1] 99.50	----------	0	0	0.12	0.29	----------	0	0.09
20	Duralumin	----------	4.1	0	[1] 94.04	----------	0	0	.31	.40	----------	.55	.60
10	Annealed Monel	----------	28.9	----------	----------	----------	----------	67.7	[2] .05	1.7	----------	1.5	----------
10a	Hard-drawn Monel	----------	26.0	----------	----------	----------	----------	70.2	[2] .05	2.1	----------	1.5	----------
2	Brass	----------	63.3	34.8	0	1.75	0	0	----------	[2] .10	----------	0	----------
8	do	----------	61.9	35.47	----------	2.6	0	0	----------	[2] .03	----------	----------	----------
7	Phosphor bronze	----------	95.8	0	----------	----------	3.9	0	----------	[2] .1	0.2	----------	----------
1	Coin silver	91.1	8.8	----------	----------	----------	----------	----------	----------	[2] .5	----------	----------	----------
6	Nickel silver	----------	58.0	26.2	0	0	----------	15.5	----------	.15	----------	.17	----------

[1] By difference.

[2] Less than amount given.

(*b*) *Ferrous*

[Elements in per cent]

No.	Samples	Carbon	Manganese	Phosphorus	Sulphur	Chromium	Vanadium	Molybdenum
11	Drill-rod steel	1.38	0.215	----------	----------	0.03	----------	----------
15	Oil-tempered steel	.70	.42	----------	----------	----------	----------	----------
14	Piano wire	.86	.294	----------	----------	----------	----------	----------
13	Chromium vanadium steel	.55	.648	----------	----------	.98	0.19	----------
18	do	.50	.64	0.02	0.03	.97	.18	----------
19	Chromium molybdenum steel	.54	.50	.02	.02	.86	----------	0.21
16	Stainless steel	.81	.129	----------	----------	13.20	----------	----------

TABLE II.—HEAT TREATMENT

No.	Sample	Quenched at ° C.	Drawn or tempered at ° C.
6	Nickel silver	----------	200, 300, 475, 650
11	Drill rod steel	785	400, 500, 564
14	Piano wire	[1] 900	----------
13	Chromium vanadium steel	925	100, 200, 300, 400
18	do	920	100
19	Chromium molybdenum steel	870	100
16	Stainless steel	1,000	200, 300, 400, 500, 600

[1] Approximate.

Dimensions of Specimens.—The effective lengths of the samples when mounted in the torsional pendulum and the average diameters are given in Table III. The diameter was measured at 4 to 6 positions on each sample, two diameters at right angles to each other at each position. The average deviations of the diameter measurements varied between the extremes of 0.1 per cent for the brass wire Number 8 and 0.8 per cent for the oil-tempered steel wire.

TABLE III.—DIMENSIONS OF SPECIMENS

(a) Nonferrous

No.	Material	Effective length, inches	Average diameter, inch	Average deviation in diameter
21	Aluminum	26.54	0.0810	0.0002
20	Duralumin	26.52	.0805	.0002
10	Annealed Monel	27.20	.0358	.0002
10a	Hard-drawn Monel	26.90	.0533	.0001
2	Brass	27.13	.0463	.0001
8	do	27.10	.0811	.0001
7	Phosphor bronze	27.10	.0816	.0005
1	Coin silver	27.11	.0329	.0001
6	Nickel silver	27.42	.0377	.0001

(b) Ferrous

No.	Material	Effective length, inches	Average diameter, inch	Average deviation in diameter
11	Drill Rod Steel:			
	As received	27.02	0.0389	----------
	After heat treatment	27.20	.0389	----------
15	Oil tempered steel: As received	27.06	.0390	0.0003
14	Piano Wire:			
	As received	26.46	.0352	.0002
	After heat treatment	----------	----------	----------
13	Chromium vanadium steel:			
	As received	27.10	.0352	.0002
	After heat treatment	27.28	.0346	.0002
18	Chromium vanadium steel:			
	As received	27.18	.0397	.0002
	After heat treatment	----------	----------	----------
19	Chromium molybdenum steel:			
	As received	27.07	.0397	.0001
	After heat treatment	----------	----------	----------
16	Stainless steel:			
	As received	26.60	.0403	.0001
	After heat treatment	27.08	.0403	.0001

DESCRIPTION OF APPARATUS AND METHODS OF PROCEDURE

Torsion Pendulum.—The torsion pendulum set up in the temperature chamber is shown in Figure 1. The wire was supported at the top by means of a chuck, the support for which included a provision for starting the swinging of the pendulum from the outside of the temperature chamber. These parts were above the cooling coils shown in the photograph. The lower chuck, near the floor of the chamber, was attached to a holder for the weights which were used to control both the tensile stress and the moment of inertia.

A platinum wire, 0.023 inch diameter, was attached to the weight holder. At the mid-point of each oscillation of the pendulum this wire made electrical contact with the free end of a fixed vertical strip of silver 0.04 by 0.002 inch in cross section and 1.5 inches long. The contact was recorded on a chronograph, by the operation of one of its two relays. The other relay was used to obtain a record of second signals received from a chronometer.

All but one of the weights were cylindrical disks, each with a hole through the center. The hole served the purpose of centering the weight on the holder. The moment of inertia of each weight was calculated from the measured dimensions and the mass. Correction was made for the slight deviations of the disks from true flatness. The moment of inertia of the chuck and holder combined was determined experimentally by the torsional pendulum method using wire specimens which had been found to have, within ¼ per cent, the same values of the torsion modulus at all of the tensile stresses applied. This method gave an accuracy of about 2 per cent in the value for the chuck and holder which is sufficient in view of the larger values of the moment of inertia of the weights as compared with that of the chuck and holder. The wire specimens used in this determination were of stainless steel and brass. Data for the weights are given in Table IV.

TABLE IV.—DATA OF WEIGHTS

Weight No.		Material	Weight, pounds	Moment of inertia in.²/lb.
3_1	Disk	Steel	3.770	22.67
3_2	do	do	3.771	22.68
3_3	do	do	3.749	22.29
5+7	Disk and ring	Brass	1.846	21.82
6	Disk	do	2.425	22.42
Holder		Steel	1.090	0.532

Temperature Chamber.—The temperature chamber is one used for testing aircraft instruments. It is considerably larger than needed for this work but is well insulated thermally by about 6 inches of cork. If the inside of the chamber is brought to −30° C. while the outside room temperature is about +20° C., the temperature within the chamber will increase to 0° C. in about 16 hours. An ammonia refrigeration system was used to cool the chamber and electric heaters to heat it. The air in the chamber was kept stirred by means of an electric fan. The fan was cut off, however, during the time when readings were being made in order to obviate a possible source of unsteadiness in the vibration. A multiple glass window in the door of the chamber and an electric light inside are part of the equipment. Temperatures were measured by a liquid-in-glass thermometer hung about 1 inch from the wire. A study of the data indicates that the error in the temperature measurements due to all causes does not exceed 1° C.

Test procedure.—In taking observations the chamber was first brought to a definite temperature and held at this value for one-half hour before obtaining data. By means of the device previously mentioned, the pendulum was then put into torsional oscillation. When the oscillations had become steady, the chronograph was started and left running until approximately 100 half oscillations had occurred. The moment of inertia, and consequently the tensile stress, was then increased by the addition of another weight to the pendulum. This operation required the opening of the chamber which affected its temperature. No observations were recorded until 15 minutes after the temperature had regained its previous value. This process was repeated until all of the observations at a given temperature were secured.

The amplitude of the oscillation was 45°, very approximately, while observations were being obtained. Data for the brass sample Number 2 and drill rod Number 11 showed that the period of both decreased one part in 2,500 per degree decrease in amplitude.

The period of oscillation of the torsion pendulum had extreme values of 1.5 seconds for phosphor bronze and

20 seconds for coin silver. For most of the specimens, ferrous and nonferrous, the period of oscillation of the pendulum had values between 4 to 10 seconds. The accuracy of the measurement of the period was of the order of 0.1 per cent.

Tensile stresses in the wires were computed from the weights given in Table IV, and the wire diameters given in Table III.

motion as the free end of the wire. The indication was on a graduated circle 7.7 inches in radius with graduations 0.25 degree apart.

Each heat-treated specimen was cut from the wire previously used in the torsional pendulum and was not independently heat-treated. The results by the two methods are therefore comparable. All tests were made with the specimen at room temperature.

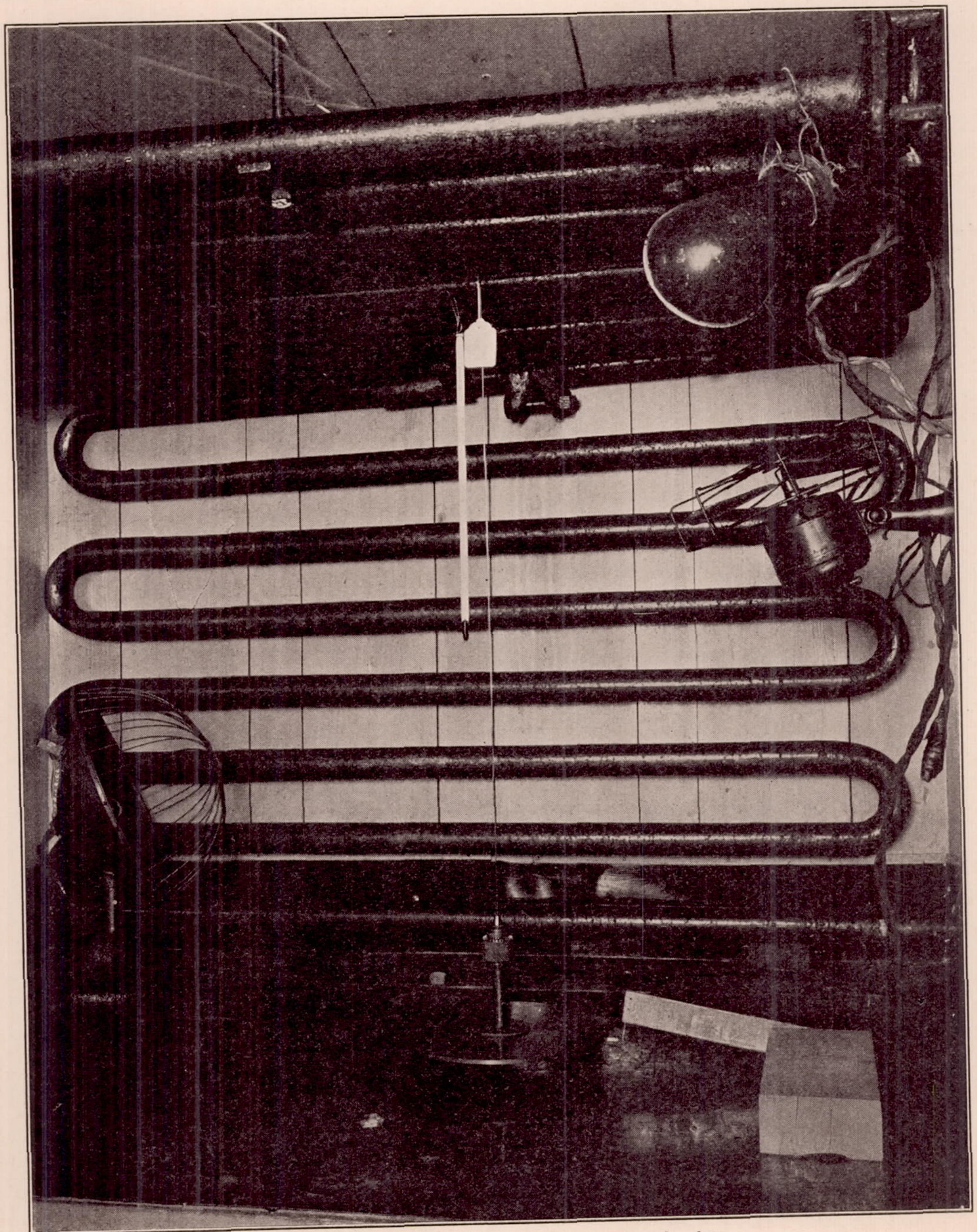


FIGURE 1.—Torsion pendulum in the temperature chamber

Determination of the Rigidity Modulus by the Deflection Method.—The apparatus is shown in Figure 3. The ends of the specimen, which is about 4 inches long, are held in pin chucks, one fixed and one free to turn. The torque is applied by means of weights in a pan, in the manner shown in Figure 3. The deflection was measured by a pointer which had the same angular

EXPERIMENTAL RESULTS

Temperature Coefficient of Rigidity Modulus.—The values of the temperature coefficient of the modulus of rigidity for the various materials are given in Table V. Two values are given for each material: One, m' uncorrected for the effect of the change in dimensions with temperature of the parts of the torsion pendulum;

and the other, m corrected for this effect. The coefficients are all negative; that is, the modulus of rigidity decreases as the temperature increases.

TABLE V.—TEMPERATURE COEFFICIENTS OF MODULUS OF RIGIDITY

No.	Material	Coefficient		Tensile stress range, lb./sq. in.
		$m' \times 10^5$ Uncorrected,[1]	$m \times 10^5$ Corrected,[1]	
21	Aluminum	−58	−62	600– 2,400
20	Duralumin	−57	−62	600– 2,400
10	Annealed Monel	−36	−38	3,000– 8,500
10a	Hard-drawn Monel	−40	−42	1,000– 6,000
2	Brass	−49	−52	1,000– 7,000
8	do	−43	−46	500– 2,400
7	Phosphor bronze	−45	−48	500– 2,300
1	Coin silver	−52	−56	2,500–11,000
6	Nickel silver			2,500–11,000
	As received	−41	−44	
	Tempered at 200° C	−39	−42	
	Tempered at 300° C	−35	−38	
	Tempered at 475° C	−33	−36	
	Tempered at 650° C	−35	−38	
11	Drill rod steel			2,000–11,000
	As received	−22	−23	
	Tempered at 400° C	−23	−24	
	Tempered at 500° C	−23	−24	
	Tempered at 564° C	−23	−24	
15	Oil tempered steel	−32	−33	2,000–11,000

[1] All values in this column to be multiplied by 10^{-5} in order to obtain m' or m.

TABLE V.—TEMPERATURE COEFFICIENTS OF MODULUS OF RIGIDITY—Continued

No.	Material	Coefficient		Tensile stress range, lb./sq. in.
		$m' \times 10^5$ Uncorrected,[1]	$m \times 10^5$ Corrected,[1]	
14	Piano wire			3,000–13,000
	As received	−36	−37	
	Quenched at approximately 900° C	−40	−42	
13	Chromium vanadium steel			3,000–13,000
	As received	−28	−29	
	Tempered at 100° C	−31	−32	
	Tempered at 200° C	−33	−34	
	Tempered at 300° C	−30	−31	
	Tempered at 400° C	−26	−27	
	Tempered at 500° C	−25	−26	
	Tempered at 625° C	−28	−29	
18	Chromium vanadium steel			2,000–10,000
	As received	−29	−30	
	Tempered at 100° C	−34	−35	
19	Chromium molybdenum steel			2,000–10,000
	As received	−47	−48	
	Tempered at 100° C	−39	−40	
16	Stainless steel			2,000–10,000
	As received	−38	−39	
	Tempered at 200° C	−31	−32	
	Tempered at 300° C	−31	−32	
	Tempered at 400° C	−31	−32	
	Tempered at 500° C	−31	−31	
	Tempered at 600° C	−22	−23	

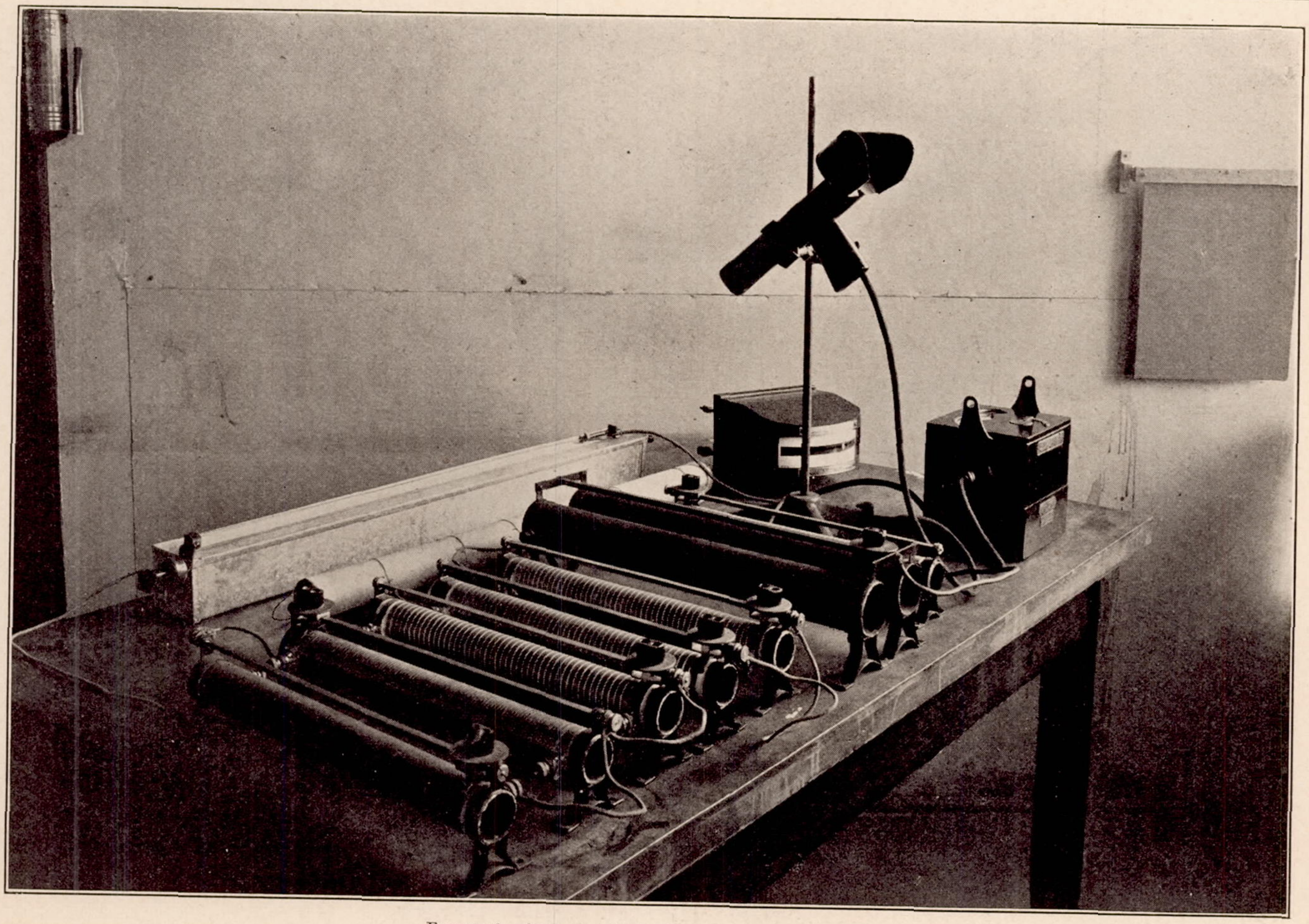


FIGURE 2.—Apparatus for hardening ferrous specimens

In the course of computing the uncorrected values graphs were drawn for all of the materials, similar to those shown in Figures 4 to 9, inclusive. These figures show the data for the specimens of nickel silver, stainless steel, and chromium vanadium Number 13.

The graphs of the expression $\frac{I}{P^2}$ against the tensile stress as shown in Figures 4, 6, and 8 were drawn first. From these the graphs of $\frac{I}{P^2}$against temperature were obtained as shown in Figures 5, 7, and 9. The values

of the uncorrected coefficients m', as defined by equation (4), are determined by the slopes of the straight lines in Figures 5, 7, and 9, divided by the values of $\frac{I}{P^2}$ at 0° C.

A careful study of the primary graphs in Figures 4, 6, and 8 led to the following two conclusions:

(1) At a given temperature $\frac{I}{P^2}$ is either constant or directly proportional to the tensile stress.

(2) For any one state of the specimen the factor of proportionality of the relation between $\frac{I}{P^2}$ and the tensile stress is independent of temperature in the temperature interval −25° to +50° C. As a consequence of (1) and (2), straight parallel lines gave the relation between $\frac{I}{P^2}$ and the tensile stress at various temperatures for a specimen in a given condition.

In a number of individual cases the observations were not in accord with the above two conclusions, for the discrepancies were greater than the experimental error in $\frac{I}{P^2}$. As an example, see the scattering of the data shown in Figure 4 for the nickel silver specimen in the "as received" condition and note that the data for −11° C. can just as well be represented by a straight line of zero slope. It will also be seen in Figure 4 that there is no such scattering of the observed points in the data obtained after the various heat treatments of the sample. It is believed that the scattering of the data for nickel silver in the "as received" condition is due to the fact that the specimen

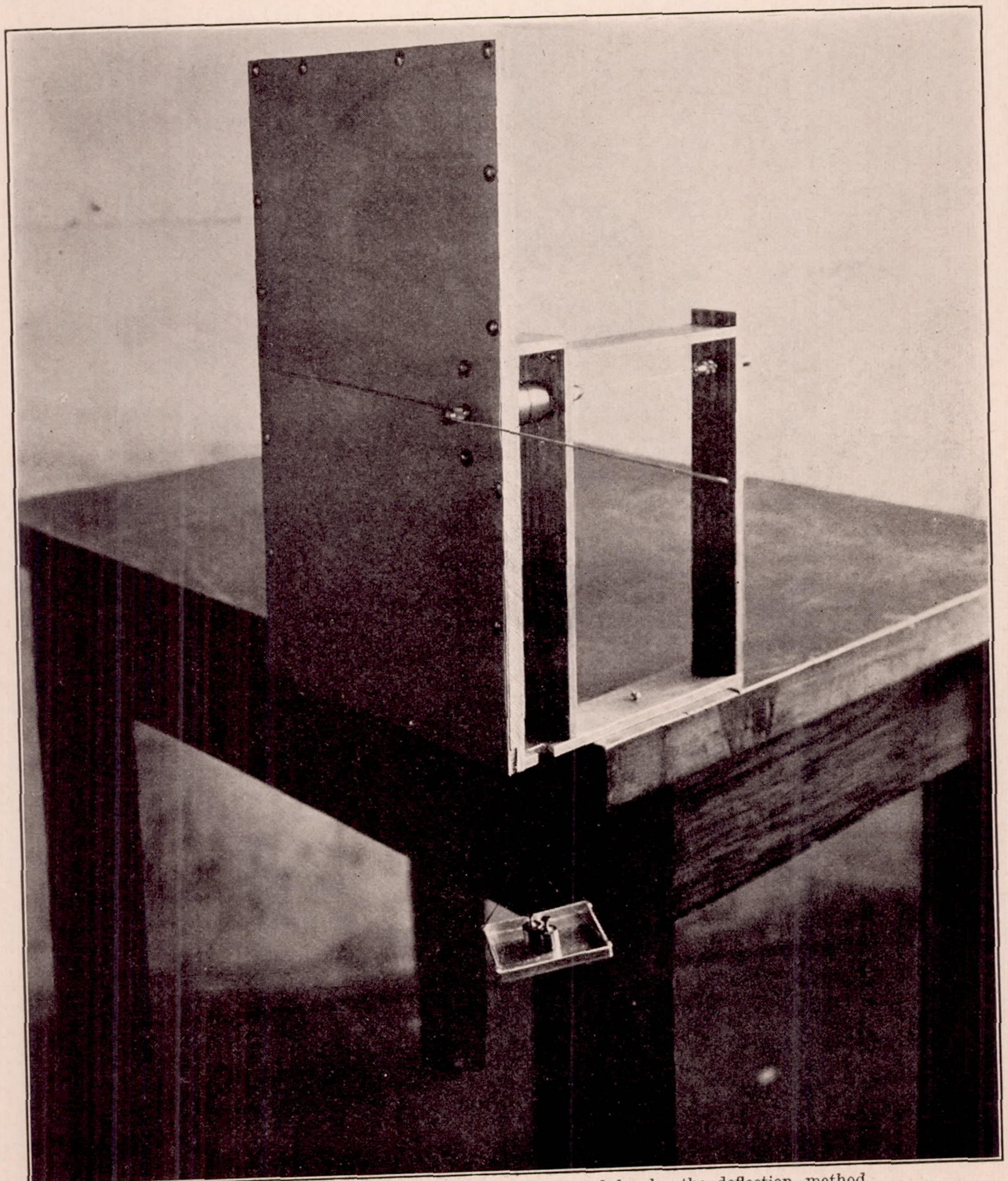


FIGURE 3.—Apparatus for determining the rigidity modulus by the deflection method

did not conform to the primary assumptions underlying formula (1). These assumptions are that the wire is straight and that it is free from internal strains. The effect of these two factors can not be separated, since the internal strains produced in drawing the wire generally cause longitudinal curvature.

Within the limits of experimental error it was found that for a given tensile stress, $\frac{I}{P^2}$ is directly proportional to temperature in the temperature interval covered by the experiments. (See Figs. 5, 7, and 9). Possible exceptions to this fact are the data for the chromium-molybdenum and annealed Monel specimens, for which

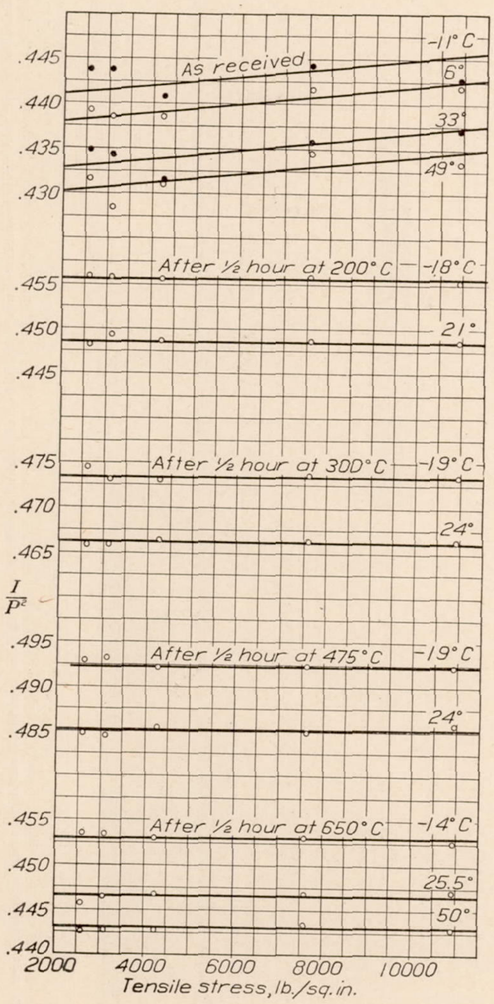


FIGURE 4.—Effect of tensile stress and temperature on the rigidity modulus of nickel silver. The moment of inertia I is in pounds-inches squared and the period P in seconds

the absolute value of the slope of the curve through the experimental points appears to increase with temperature. However, the best straight line was drawn through the points for these materials also, since the deviation does not greatly exceed the experimental error. This point is considered later in greater detail.

The coefficient m varies less than 1 per cent with tension over the range of tensile stresses applied.

The values of m, the temperature coefficient corrected for the expansion of the parts of the torsion pendulum, were computed by means of equation (11). It should be noted in the computation of m that all of the experimental values of m' are negative in sign.

Modulus of Rigidity at 0° C.—The values of the modulus of rigidity for one tensile stress are given in

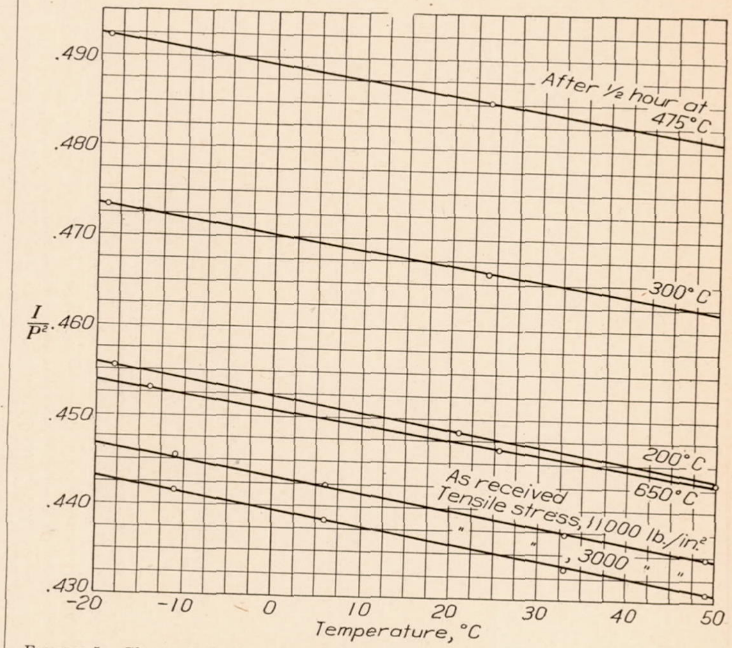


FIGURE 5.—Change in the rigidity modulus of nickel silver with temperature after various heat treatments. The moment of inertia I is in pounds-inches squared and the period P in seconds

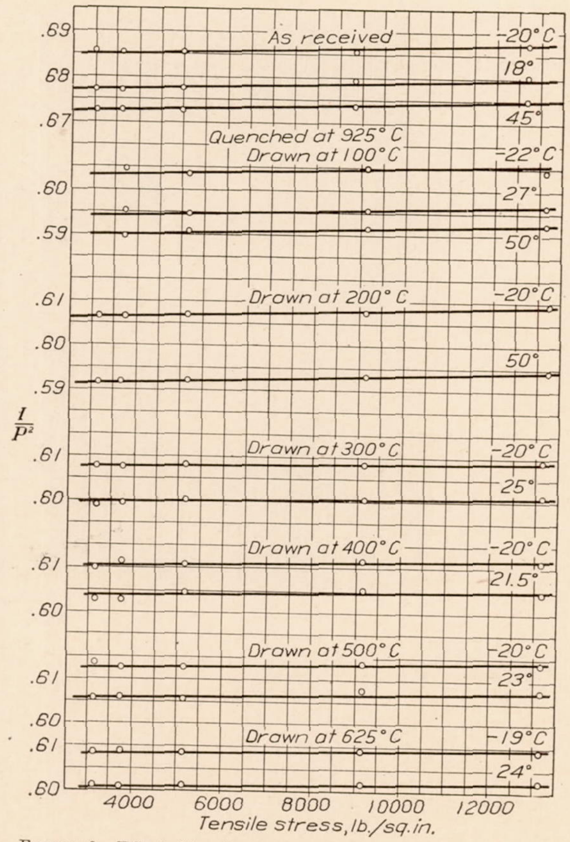


FIGURE 6.—Effect of tensile stress, heat treatment, and temperature on the rigidity modulus of chrome vanadium steel. The moment of inertia I is in pounds-inches squared and the period P in seconds

Table VI. The table gives also the percentage change in modulus for change of tensile stress in a given range of stress.

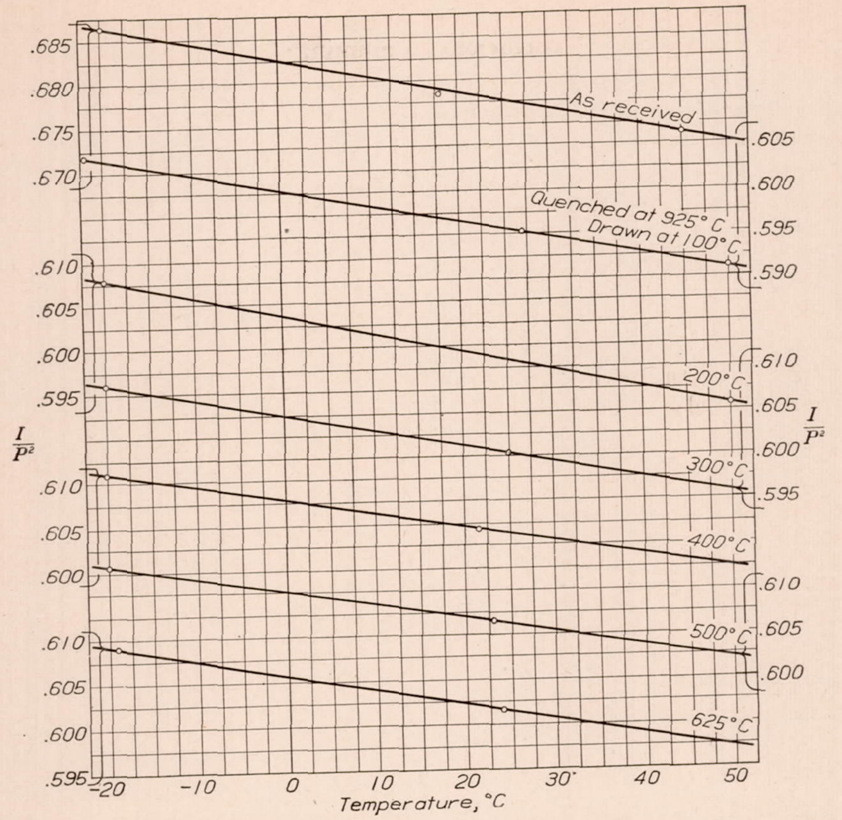


FIGURE 7.—Change in the rigidity modulus of chrome vanadium steel with temperature after various heat treatments. The moment of inertia I is in pounds-inches squared and the period P in seconds

TABLE VI.—MODULUS OF RIGIDITY AND EFFECT OF TENSILE STRESS

No.	Material	Rigidity modulus at 0° C. 10^6 lb./sq. in. — Tensile stress lb./sq. in. 1,000	5,000	8,000	Increase in modulus with tensile stress — Stress range lb./sq. in.	Increase, per cent
21	Aluminum	3.74			600– 2,400	0
20	Duralumin	3.94			600– 2,400	.54
10	Annealed Monel		9.68		1,000– 6,000	.45
10a	Hard drawn Monel		9.85		1,000– 7,000	.52
2	Brass		5.03		1,000– 7,000	.56
8	do	5.03			500– 2,400	.39
7	Phosphor bronze	6.42			500– 2,300	.41
1	Coin silver		4.25		2,500–11,000	−.62
6	Nickel silver				2,500–11,000	
	As received		6.25			.97
	Tempered at 200° C		6.42			0
	Tempered at 300° C		6.67			0
	Tempered at 475° C		6.94			0
	Tempered at 650° C		6.40			0
11	Drill rod steel				2,000–11,000	
	As received		11.2			.92
	Tempered at 400°C		11.1			.48
	Tempered at 500° C		11.1			.48
	Tempered at 600° C		11.1			.48
15	Oil tempered steel			11.3	3,000–11,000	.43
14	Piano wire				3,000–13,000	
	As received			11.4		.23
	Quenched at about 900° C					.21
13	Chromium vanadium steel				3,000–13,000	
	As received			12.6		.40
	Tempered at 100° C			11.9		.44
	Tempered at 200° C			12.0		.44
	Tempered at 300° C			12.0		.20
	Tempered at 400° C			12.1		.20
	Tempered at 500° C			12.1		.20
	Tempered at 625° C			12.0		.20
18	Chromium vanadium steel				2,000–10,000	
	As received			11.4		0
	Tempered at 100° C					−.34
19	Chromium molybdenum steel				2,000–10,000	
	As received		11.7			−.20
	Tempered at 100° C					.12
16	Stainless steel				2,000–10,000	
	As received	12.6	12.6			0
	Tempered at 200° C		12.3			.18
	Tempered at 300° C		12.3			.18
	Tempered at 400° C		12.3			.18
	Tempered at 500° C		12.4			.18
	Tempered at 600° C		12.5			.18

The modulus of rigidity was computed using formula (3). The value of $\frac{I}{P^2}$ was obtained from graphs such as Figures 5, 7, and 9. The length and radius of the wire specimens are given in Table I. No correction was made for the effect of temperature on the dimensions of the parts of the torsion pendulum since its amount in no case exceeds 0.1 per cent.

The error in the values of the modulus introduced by neglect of the effect of damping was calculated for brass samples Numbers 2 and 8, and drill rod sample Number 11, based on experimental data. It was found to be negligible (less than 10^{-3} per cent). Since the data were obtained in the usual manner, it indicates that the work done by the vibrating system in deflecting the contact strip was very slight.

Microphotographs were made of three groups of chromium vanadium wires of the lot from which specimen Number 18 was taken in order to determine the extent to which decarburization occurred during the heat treatment. One sample was quenched from 920° C. in water, the second similarly quenched and then tempered at 600° C., and the third was as received. The heat treatment was given in the apparatus previously described. No evidence of decarburization was found in any of the three samples. However, the two groups of samples which had been quenched from 920° C. were found to have cracks, approximately radial, extending almost to the center of the wire. The effect of the cracks probably reduces

the apparent value of the modulus, but is not believed to affect the value of the temperature coefficient. This offers an explanation of the differences in the modulus for chromium vanadium specimen Number 13 in the "as received" condition and after heat treatment (about 5 per cent).

Comparison of Moduli Values by Deflection and Pendulum Methods.—The modulus of rigidity was also computed from the data obtained by the deflection method using formula (12). In each case the

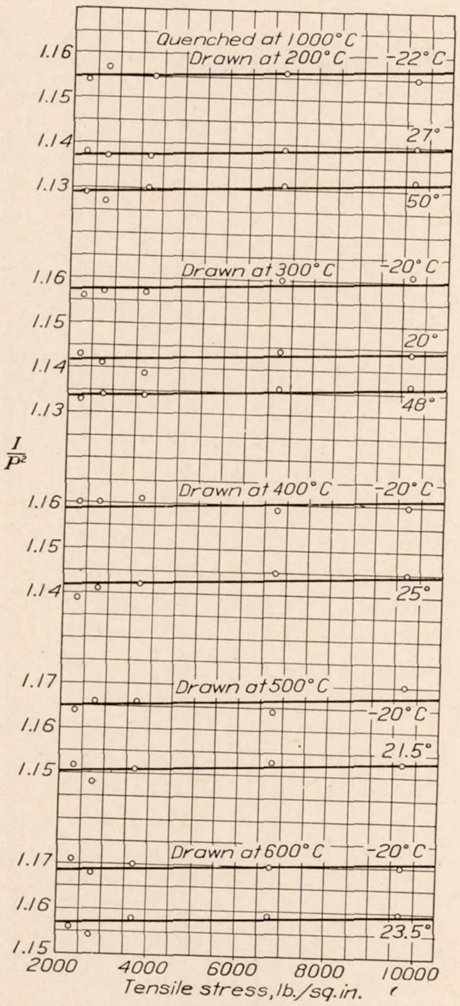


FIGURE 8.—Effect of tensile stress and temperature on the rigidity modulus of stainless steel. The moment of inertia I is in pounds-inches squared and the period P in seconds

term M/a of formula (12) was determined by computing the slope of the best straight line which was drawn through the torque-deflection data. The results of these computations are given in Table VII, together with values comparable as to tensile stress and temperature which were determined by means of the torsion pendulum. In order to secure the latter values at zero tensile stress the straight lines of graphs such as given in Figures 4, 6, or 8 were extrapolated. The accuracy of the determinations by the deflection method is estimated to be 2 per cent. The agreement is, in general, satisfactory. It is seen that the values by deflection are the smaller except for aluminum, duralumin, and piano wire. The difference for the latter material is within the experimental error.

TABLE VII.—COMPARISON OF DATA OBTAINED FROM DEFLECTION AND PENDULUM TESTS

[The values of the modulus from the torsion pendulum given below are for zero stress in tension and have been extrapolated from the data]

No.	Material	Temperature of specimen, ° C.	Rigidity modulus in 10^6 lb./sq. in.		Maximum shear stresses, 10^3 lb./sq. in.	
			Deflection method	Pendulum method	Deflection tests	Pendulum tests
21	Aluminum	+27	3.9	3.69	4.5	10
20	Duralumin	26	4.0	3.86	5	11
10	Annealed Monel	28	9.3	9.60	16	11
10a	Hard-drawn Monel	28	9.5	9.68	8	17.5
2	Brass	28	4.75	4.94	7	8
8	do	29	4.9	4.95	4.5	14
7	Phosphor bronze	27	6.2	6.32	4.5	18
1	Coin silver	28	4.3	4.32	13.5	5
6	Nickel silver (heat-treated at 650° C)	27	6.2	6.34	13	8
11	Drill rod steel	---	---	---	---	15
13	Chromium vanadium	---	---	---	---	14
18	do	28	11.2	11.4	39	15
15	Oil tempered steel	30	10.9	11.1	39	15
14	Piano wire	29	11.4	11.3	54	14
16	Stainless steel	27	12.4	12.5	10.5	16.5
19	Chromium molybdenum	28	10.8	11.5	38	15

Effect of Tensile Stress on Modulus.—The change in the modulus of rigidity for the range of tensile stress for which data were obtained is given in Table VI. No great accuracy is claimed for the data, which are presented merely as qualitative evidence that the effect exists. See later discussion of results for phosphor bronze.

Maximum Shear Stresses.—The maximum stress in shear to which the specimens were subjected both during oscillation of the pendulum and in the deflection tests was computed from the formula

$$S = \frac{Ar}{l} G \tag{13}$$

in which S is the stress at angular deflection A, and r and l are the radius and length of the wire, respectively. These values are given in Table VII. Those for the pendulum specimens are for an amplitude of 0.9 radian.

DISCUSSION

It should be emphasized that the absolute values of the modulus may be in error due to discontinuities in the surface of the wires. This is an important factor in the case of the ferrous materials. On the other hand, it is believed that the temperature coefficient values are of greater reliability since they depend upon measurements of the change in the modulus. It is assumed that the effect of the internal stresses and discontinuities in the wire surface do not vary with temperature.

In general, other investigators using the torsion pendulum, notably Iokibé and Sakai, Horton, and those preceding Horton, obtained data only on annealed specimens. In the absence of a definite statement it is

not certain whether the specimens of Koch and Dannecker were annealed or not, but the data are characteristic of those obtained on hardened samples.

Horton and Chevenard correct their results for the effect of the change in dimensions of the specimen and dependent parts with temperature. It is not certain of the coefficient to increase slightly with temperature. The modulus of rigidity is computed from their data to be 3.69×10^6 pounds per square inch at 0° C.

Koch and Dannecker's data (Reference 9) give -61×10^5 for the average value of the temperature coefficient of the modulus of rigidity in the tem-

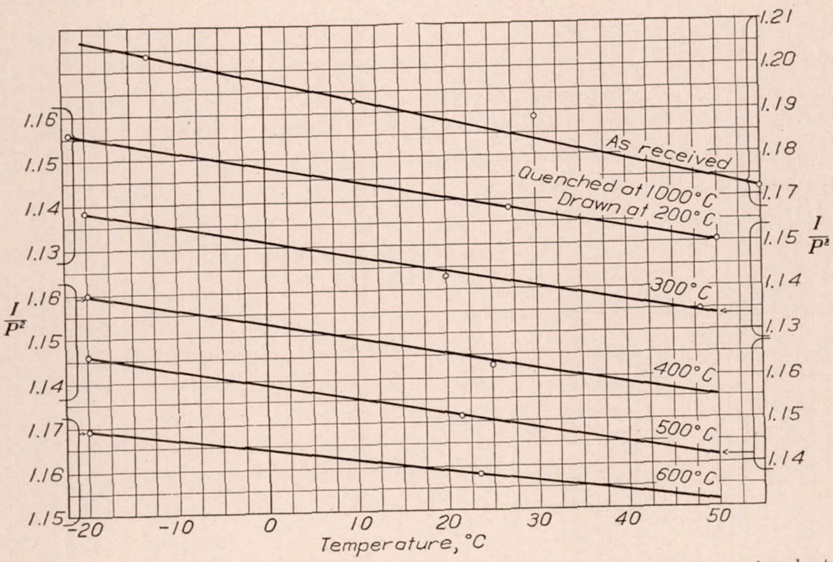


FIGURE 9.—Change in the modulus of rigidity of stainless steel with temperature after various heat treatments. The moment of inertia I is in pounds-inches squared and the period P in seconds

that the results of Koch and Dannecker or Iokibé and Sakai are so corrected.

The available data for aluminum and Monel metal indicate that the temperature coefficients of elasticity are independent of temperature within the experimental error when these materials are in the hard-drawn condition, but vary with temperature when annealed. All of the nonferrous metals tested in the hard-drawn condition had values independent of temperature. It is offered as a tentative conclusion that hard-drawn and annealed nonferrous metals generally show this difference in the temperature coefficients. Further details are given below.

No conclusion can be drawn from the available data as to the difference in dependence upon temperature of the temperature coefficients of annealed and hardened ferrous metals.

Aluminum.—Horton (Reference 1) found that the modulus of rigidity of an annealed wire varied irregularly with heat treatment in the temperature range from room temperature to 100° C. He reports the temperature coefficient to be -135×10^{-5} at 15° C. and the modulus of rigidity to be 3.73×10^6 pounds per square inch at 0° C. The latter value is computed from the value at 15° C.

The data of Iokibé and Sakai (Reference 3) for an annealed aluminum wire give an average value of -103×10^{-5} for the temperature coefficient of the modulus of rigidity in the temperature interval +25° to 76° C. Their data show the numerical value perature interval +20° to 100° C., which agrees closely with the value given in Table V (-62×10^{-5}), and 3.96×10^6 pounds per square inch for the value of the modulus at 0° C. The condition of their wire specimens, whether initially hard or annealed, is not stated. The values for the modulus found by Horton and in our experiments (3.74×10^6) agree closely but

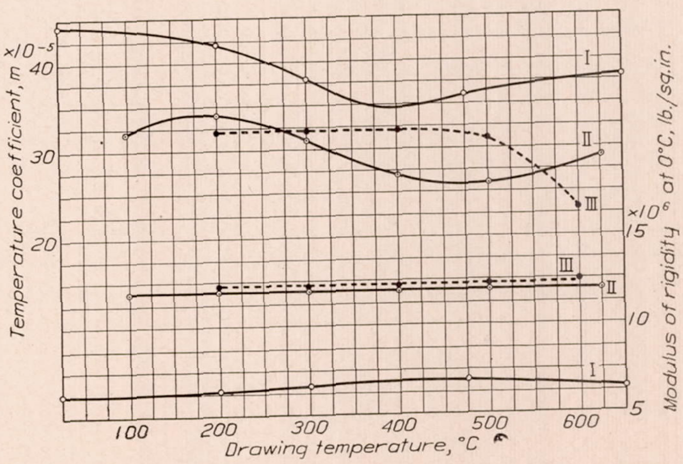


FIGURE 10.—Effect of tempering or drawing temperature upon the temperature coefficient and modulus of rigidity. Curves I are for nickel silver, curves II for chromium vanadium steel and curves III for stainless steel

are much less than those reported by Koch and Dannecker.

Dodge (Reference 8) experimented with both a hardened and an annealed aluminum specimen and found that the temperature coefficient of Young's modulus of the hardened specimen was substantially

constant up to 100° C., but that of the annealed one varied with temperature. This difference in behavior of annealed and hardened aluminum may explain the variation in values reported by us and by Horton and others. An objection to this explanation is that Dodge's value for the temperature coefficient of Young's modulus of the hard specimen, -122×10^{-5} at 20° C., is greater than for the annealed one, -61×10^{-5} at 20° C., while a comparison of Horton's and our data shows the temperature coefficient of the modulus of rigidity of the harder specimen to be less.

Koch and Dieterle (Reference 11) obtained an average value of the temperature coefficient of Young's modulus for an annealed specimen of -65×10^{-5} in the temperature interval +17 to 110° C. This is in agreement with Dodge.

It may be safely concluded from a consideration of all of the data that heat treatment at relatively low temperatures greatly affects the temperature coefficients of elasticity of aluminum. The effect is far greater than for other metals considered in this report.

The data of all of the investigators mentioned above show fairly conclusively that the numerical value of the temperature coefficient of the modulus of rigidity of annealed aluminum increases slightly with temperature and that it is constant for hardened aluminum up to about 100° C.

Duralumin.—Koch and Dannecker (Reference 9) gave values of the modulus of rigidity at +20° and 100° C., which give an average value of the temperature coefficient of -46×10^{-5} in this temperature range. This appears comparable with the value in Table V (-62×10^{-5}), since both data show the variation of the modulus with temperature to be uniform in this temperature interval. A source of discrepancy in the two values is the probable difference in the initial state of the specimens. There appears to be no great variation due to heat treatment or other causes from the value given in Table V, such as is the case for aluminum.

The modulus of rigidity at +20° C. given in the above reference is 3.87×10^6 pounds per square inch from which is computed the value at 0° C., 3.91×10^6 pounds per square inch. This is in close agreement with the value 3.94×10^6 given in Table VI.

Monel Metal.—The hardened specimen has a somewhat higher modulus of rigidity and larger average temperature coefficient than the annealed specimen but no great significance should be attached to the difference.

A more significant difference is the fact that the value of the temperature coefficient is constant for the hardened specimen (-42×10^{-5}) in the temperature range −14° to +50° C., while on the other hand, it varies from -21×10^{-5} at −13° C. to -54×10^{-3} at +50° C. for the annealed specimen. These values are estimated to be accurate within 10 per cent. This difference in behavior of the annealed and hard specimens is similar to that reported by Dodge for aluminum (Reference 8).

The modulus of rigidity is given as 9.5×10^6 pounds per square inch by the International Nickel Co., which probably is an average value at a temperature of about +20° C. This compares with values here reported of 9.68 and 9.85 pounds per square inch.

Brass.—Koch and Dannecker (Reference 9) give a value of -42×10^{-5} for the temperature coefficient of the modulus of rigidity in the temperature range +20 to 100° C. for a brass composed of 60 per cent copper and 40 per cent zinc. For a specimen of duranametal, supposed by them to be of about the same composition as their brass specimen except for the addition of small unknown amounts of other metals, the temperature coefficient is -35×10^{-5} between +20 and 100° C. Their data indicate that the coefficient is independent of temperature to the first order in the temperature interval +20 to 100° C.

The values of the temperature coefficient found in the present experiments are −46 and -52×10^{-5} for the two samples of brass. The difference in the temperature coefficients is not easily explained at first sight since the absolute values of the moduli of rigidity were found to be alike, and since it seems reasonable to suppose that physical differences in the two samples would affect the modulus as well as the coefficient. It is believed, however, that the agreement in the two values of the modulus is accidental and that the actual values differ, a conclusion which appears to be confirmed by some additional data obtained on brass sample Number 2 at +30° C., which gave a value of the modulus about 3 per cent greater than that given in Table V. It is further to be noted that the additional data at 30° C. is independent of the stress in tension.

It is of interest to note that the data of Koch and Dieterle (Reference 11) give -38×10^{-5} for the average temperature coefficient of Young's modulus in the temperature interval +11 to 98° C.

The modulus of rigidity at 0° C. is 5.59 and 5.55×10^6 pounds per square inch, respectively, for the two brasses considered by Koch and Dannecker. These are to be compared with the values 5.03×10^6 given in Table VI.

Phosphor Bronze.—The average value of the temperature coefficient is computed to be -34×10^{-5} in the temperature interval +20 to 100° C. and the modulus of rigidity, 5.32×10^6 pounds per square inch at 0° C. from the data of Koch and Dannecker (Reference 9). Their data show the absolute value of the temperature coefficient to increase with temperature. The composition of their specimen was 93 per cent copper and 7 per cent tin. The results do not appear comparable with those here reported, i. e., -48×10^{-5} for the temperature coefficient and 6.42×10^6 pounds

per square inch for the modulus. This discrepancy may be due to differences in chemical composition and hardness. Their value for the modulus of rigidity appears to be very low for a phosphor bronze.

St. Clair (Reference 2) has given data on the temperature coefficient of Young's modulus based on measurements of the deflection of hair springs which give a value of -38×10^{-5} for the coefficient. The phosphor bronze was of the grade most suitable for use in diaphragms and springs and was in the hardened condition.

Edwards, Bowen, and Alty (Reference 5) have investigated the effect of stress in tension on the values of the modulus of rigidity of phosphor bronze when measured by means of the torsion pendulum. This effect was first pointed out by Pealing. Their experiments indicate that the modulus increases with tensile stress up to a value of about 1,200 to 1,500 pounds per square inch, and for values above this stress is substantially constant. The effect is greater in hardened specimens and is reduced by annealing. The data here reported indicate an increase of 0.41 per cent in the modulus from a tensile stress of 500 to one of 2,300 pounds per square inch. This increase has been assumed to hold good at all of the temperatures at which readings were made. No great significance can be attached to the value for the effect found in the present experiments since the average deviation of the points from the straight lines which give the relation between the tensile stress and $\frac{I}{P^2}$ is too great. The data are poorer in this respect than for any other material. Another point is that the present experiments have not been conducted so as to bring out this effect, although there is definite evidence that the modulus of rigidity of phosphor bronze and of many of the materials tested depends on the tensile stress.

Coin Silver.—There appear to be no available data on the elastic constants of this material. However, data on silver may be used as a rough basis of comparison.

The values of the constants for silver derived from the data of other investigators are given below:

SILVER

Investigators	Modulus of rigidity at 0° C., 10^6 lb./sq. in.	Temperature coefficient	
		Average value$\times10^5$	Temperature interval, °C.
Horton	3.90	−48 −45	0 to 100, at 15° C.
Koch and Dannecker	3.87	−44	0 to 100.
Iokibé and Sakai	3.25	−57	+29 to 92.

The modulus of rigidity of coin silver here reported is 4.25 pounds per square inch at 0° C. and the temperature coefficient, -56×10^{-5}.

Nickel Silver.—The average temperature coefficient of the modulus of rigidity, derived from Koch and Dannecker's data, is -39×10^{-5} in the temperature range +20 to 100° C. and the modulus is 6.85×10^6 pounds per square inch at 0° C. There is no information on the exact composition of their specimens. The above values agree reasonably well with those of the present data for the specimen tempered at 300° C. The values of the temperature coefficient and of the modulus are plotted against the tempering temperatures in Figure 10. A study of the effect of tempering at various temperatures by Thompson and Whitehead (Reference 6) indicates a change in the internal structure of nickel silver with 15 per cent nickel when tempered in the temperature region 300 to 400° C. and, less certainly, at a tempering temperature of 550° C. The data of Figure 10 do not contradict this conclusion but do not necessarily corroborate it. A change in structure is clearly indicated in both curves of Figure 10 somewhere between 300 and 475° C., but owing to lack of data in this interval the exact temperature is not indicated. If a change in structure occurs at 500 to 600° C., it is apparently without effect on the temperature coefficient or the modulus.

Figure 4 shows a distinct difference in behavior of the specimen before and after heat treatment which is believed to be typical for cold-drawn and heat-treated wires of nonferrous materials. The experimental points deviate greatly and in an erratic manner from the best straight line for the cold-drawn sample, which deviation all but disappears after the specimen has been subjected to a temperature of 200° C. The fact that the modulus is independent of tension after heat treatment is not believed to be necessarily true for the other nonferrous materials. The deviation of individual points from a straight line for the hard-drawn specimens of other nonferrous materials is of about the same magnitude. This is in general agreement with the observations of other observers who state that it was necessary to anneal the specimens in order to secure consistent data.

Drill Rod Steel.—Iokibé and Sakai (Reference 3) give data on a steel containing 1.3 per cent carbon which is substantially the same as that of the drill rod sample. According to their results, the average temperature coefficient of the modulus of torsion is zero in the temperature interval +29° C. to 77° C. and the modulus is 11.0×10^6 pounds per square inch at 0° C. The temperature coefficient is constant in this temperature range. The value for the modulus is in agreement with that of the present experiments, but there is disagreement on the values of the temperature coefficient.

Oil Tempered Steel.—No other data appear to have been published on this material.

Piano Wire.—Horton finds the temperature coefficient of rigidity of two annealed specimens of "pianoforte wire" to be -26×10^{-5} in the temperature interval +10 to 100° C. and to be constant in this temperature interval. The modulus of rigidity is given as 12.3 and 11.9×10^6 pounds per square inch for the two samples.

From the data of Iokibé and Sakai the average temperature coefficient for an annealed sample of steel containing 0.90 per cent carbon is found to be

-11×10^{-5} in the temperature interval $+56$ to 104° C., and the modulus of rigidity to be 11.1×10^{6} pounds per square inch.

Chromium Vanadium Steel.—Figure 10 shows that the temperature coefficient of the modulus is greatly affected by the tempering temperature, and that the modulus of rigidity is unaffected. The absolute values of the modulus as determined subsequent to quenching are undoubtedly too small due to quenching cracks as has been previously stated.

Chromium Molybdenum Steel.—The temperature coefficient (-39×10^{-5}) for this material, determined after the heat treatment given in Table V, is an average value for the temperature interval -15 to $+50$° C. The average value in the interval -15 to 0° C. is -21×10^{-5}, and in the interval $+25$ to 50° C. is -45×10^{-5}.

Stainless Steel.—The remarkable constancy of the modulus and the temperature coefficient shown in Figure 10 for tempering temperatures up to 500° C. is analogous to the behavior of other properties of stainless steel. See Reference 10 in this connection. A change in the crystal structure takes place at a temperature somewhat above 500° C., which accounts for the change in the temperature coefficient after tempering at 600° C.

No published data were found on the temperature coefficient.

CONCLUSIONS

The values of the temperature coefficient of the modulus of rigidity have been determined for a number of nonferrous and ferrous metals, in the temperature range -20 to $+50$° C. The values are given in Table V.

Values of the modulus of rigidity were also measured. These are given in Table VI.

The effects of tension and of heat treatment were considered. These data are summarized in Tables V and VI.

Table VIII summarizes the present state of knowledge on the temperature coefficient of the modulus of rigidity of the materials considered in this paper for the temperature range -20 to $+50$° C. The results of all investigators of these materials whose work has come to our attention were given due weight in determining the values given. The values are tentative in many cases.

TABLE VIII.—TEMPERATURE COEFFICIENT OF THE MODULUS OF RIGIDITY IN THE TEMPERATURE RANGE -20 TO $+50$° C.

Material	Composition, per cent	Condition of specimen	Temperature coefficient,[1] $\times10^{5}$
Aluminum		Annealed	−100 to −135
	Al 99.5	Half-hard	−62
Duralumin	Cu 4.1	Heat-treated	−62
		Unknown	−46
Monel	Cu 26, Ni 70.2	Hard-drawn	−42
	Cu 28.9, Ni 67.7	Annealed	−38
Brass	Cu 63, Zn 35, Pb 1.8	Hard-drawn	−52
	Cu 62, Zn 35, Pb 2.6	do	−46
	Cu 60, Zn 40	Unknown	−40
Phosphor bronze	Sn 3.9	Hard-drawn	−48
	Sn 7	Unknown	−34
Coin silver	Cu 8.8	Hard-drawn	−56
Nickel silver	Cu 57, Zn 26, Ni 15.5	do	−44
	do	Heat-treated	−36 to −42
	do	Tempered at 200° C.	−42
Drill rod steel	C 1.38	Heat-treated	−23 to −24
Oil tempered steel	C 0.70	Heat-treated spring wire.	−33
Piano wire	C 0.86	do	−37
	do	Hardened	−42
		Annealed	−26
Chromium vanadium steel.	C 0.55	Heat-treated	−26 to −34
	do	Quenched, tempered at 300° C.	−31
	C 0.50	Unknown	−30
	do	Hardened	−35
Chromium molybdenum steel.	C 0.54	Unknown	−48
	do	Hardened	−40
Stainless steel	C 0.81	Quenched, tempered at 200° to 500° C.	−31 to −32
	do	Quenched, tempered at 600° C.	−23

[1] All values in this column to be multiplied by 10^{-5}.

ACKNOWLEDGMENTS

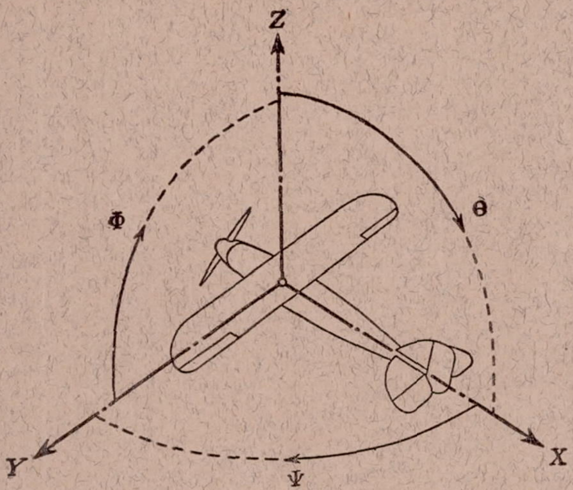
It is desired to acknowledge the cooperation of the Aluminum Co. of America, the American Steel and Wire Co., Stewart, Hartshorn & Co., and of the Foxboro Co. in furnishing specimens of various materials. The assistance of Messrs. R. B. Block and M. R. Houseman in the computations has been appreciated.

BUREAU OF STANDARDS,
WASHINGTON, D. C., *March 7, 1930.*

REFERENCES

1. Horton, Frank: The Effects of Changes of Temperature on the Modulus of Torsional Rigidity of Metal Wires. Phil. Trans. Roy. Soc. V–204A, 1905, p. 1–55.
2. St. Clair, B. W.: Springs for Electrical Measuring Instruments. Mech. Eng., p. 1057–1058, V. 47, 1925.
3. Iokibé, K., and Sakai, S.: The Effect of Temperature on the Modulus of Rigidity, and the Viscosity of Solid Metals. Science Reports, Tohoku Imperial University, p. 1–27, V. 10, 1921.
4. Chevenard, P.: Recherches experimentales sur les alliages de fer, de nickel et de chrome. Travaux et Memoires de Bur. Int. des Poids et Mes. p. 1–142, V. 17, 1927.
5. Edwards, E., Bowen, I., and Alty, S.: The Effect of Tension Certain Elastic Properties of Wires. Phil. Mag., p. 321–340, V. 2, 1926.
6. Thompson, F. C., and Whitehead, E.: Some Mechanical Properties of the Nickel-Silvers. Journ. Inst. of Metals, p. 227–259, V. 27, 1922.
7. Guillaume, C.-Ed.: Recherches. Metrologiques sur les aciers an nickel. Travaux et Memoires de Bur. Int. des Poids et Mesures, p. 1–240, V. 17, 1927.
8. Dodge, H. L.: Change in Elasticity of Aluminum Wire. Phys. Rev., p. 312–318, V. 6, 1915.
9. Koch, K. A., and Dannecker, C.: Die Elastizitaet einiger Metalle und Legierungen bis zu Temperaturen, die ihrem Schmelzpunkt naheliegen. A. Torsionselastizitaet. Ann. d. Phys., p. 197–227, V. 47–4, 1915.
10. Monypenny, J. H. G.: Resistance to Corrosion of Stainless Steel and Iron. Trans. Faraday, Soc. p. 169, V. 19, 1923.
11. Koch, K. R., and Dieterle, R.: Die Elastizitaet einiger Metalle und Legierungen bis zu Temperaturen, die ihrem Schmelzpunkt Naheliegen. B. Elastizitaetsmodul. Ann. d. Physik, p. 441–462, V. 68, 1922.

U. S. GOVERNMENT PRINTING OFFICE: 1930



Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	rolling	L	$Y \longrightarrow Z$	roll	Φ	u	p
Lateral	Y	Y	pitching	M	$Z \longrightarrow X$	pitch	Θ	v	q
Normal	Z	Z	yawing	N	$X \longrightarrow Y$	yaw	Ψ	w	r

Absolute coefficients of moment

$$C_L=\frac{L}{qbS} \qquad C_M=\frac{M}{qcS} \qquad C_N=\frac{N}{qfS}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter.
p_e, Effective pitch.
p_g, Mean geometric pitch.
p_s, Standard pitch.
p_v, Zero thrust.
p_a, Zero torque.
p/D, Pitch ratio.
V', Inflow velocity.
V_s, Slip stream velocity.

T, Thrust.
Q, Torque.
P, Power.
(If "coefficients" are introduced all units used must be consistent.)
η, Efficiency $= T\,V/P$.
n, Revolutions per sec., r. p. s.
N, Revolutions per minute, r. p. m.
Φ, Effective helix angle $=\tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp = 76.04 kg/m/s = 550 lb./ft./sec.
1 kg/m/s = 0.01315 hp
1 mi./hr. = 0.44704 m/s
1 m/s = 2.23693 mi./hr.

1 lb. = 0.4535924277 kg
1 kg = 2.2046224 lb.
1 mi. = 1609.35 m = 5280 ft
1 m = 3.2808333 ft.